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Instability in the voluntary contribution mechanism with a quasi-linear payoff function: An experimental analysis

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A B S T R A C T

We conduct experiments to investigate the convergence of contributions in the voluntary contribution mechanism (VCM) with two quasi-linear payoff functions. One is linear with respect to private goods and nonlinear with respect to public goods; we call it “QL1.” The other is linear with respect to public goods and nonlinear with respect to private goods; we call it “QL2.” The system with QL1, built on the assumption of self-interested players and myopic Cournot best response dynamics, is not stable, but the system with QL2 has a dominant Nash equilibrium. This theoretical result predicts a “pulsing” of contributions in the VCM with QL1. Our experimental observations demonstrate that individual contributions are certainly converging to the dominant Nash equilibrium in the experiment with QL2. In the experiments with QL1, however, the dispersion of individual contributions increases progressively with repeated trials, and the contributions are still volatile in the experiments' last periods, although we do not find a clearly unstable pulsing in the group's total contribution.

1. Introduction

The voluntary contribution mechanism (VCM) has been investigated by experimental economists for many years in order to understand the public goods provision problem.[1] Most researchers in this field use linear payoff functions such as $u(x_i, y) = x_i + by$, where x_i is a private good of player i, y is a public good, and b is a positive constant. However, several scholars argue that this linear payoff setting cannot represent real-world situations of the VCM environment because the self-interested choice (Nash equilibrium) and the optimal social choice are located at opposite boundaries of the feasible choice set (see, e.g., Sefton and Steinberg, 1996; Laury and Holt, 2008).

One way to address this problem is to adopt nonlinear payoff functions to provide an interior solution for the self-interested choice and the optimal social choice. Thus, two quasi-linear payoff functions are introduced in the literature. The economic rationale of the first payoff function is that private good x_i is money; therefore, its marginal return could be assumed to be constant. However, the marginal return from specific public good *y* is nonlinear. That is, $\pi(x_i, y) = x_i + t(y)$ (see Isaac et al., 1985; Isaac and Walker, 1991; Sefton and Steinberg, 1996; Isaac and Walker, 1998; Laury et al., 1999; Hichri and Kirman, 2007). We call this “QL1.” Conversely, the second payoff function is linear with respect to y and nonlinear with respect to x_i. Thus, the function is $\pi(x_i, y) = h(x_i) + y$ (see Sefton and Steinberg, 1996; Keser, 1996; Falkinger et al., 2000; Willinger and Ziegelmeyer, 2001; van Dijk et al., 2002; Uler, 2011; Maurice et al., 2013; Cason and Gangadharan, 2014). We call this “QL2.” The second payoff function is used to model a relatively rare situation in which the marginal return from the private good decreases, whereas it is constant for the public good.

These two designs lead to completely different theoretical predictions. The VCM with QL1 induces multiple static Nash equilibria, which produces a coordination problem. By contrast, the VCM with QL2 induces a unique dominant equilibrium, which is similar to the VCM with linear payoff functions. Sefton and Steinberg (1996) compared contribution levels across QL1 and QL2 environments using a randomly rematched group setting to suppress the feedback from the results of previous periods in the experiments. They predicted that the presence of the coordination problem should partially explain why the average of individual contributions is significantly above the Nash prediction in their design of the VCM with QL1, although their experimental results indicated only a slight difference in contribution levels between the two experiments.

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[1] See Ledyard (1995) and Chaudhuri (2011) for surveys on experiments regarding the VCM. Bergstrom et al. (1986) discuss the basic theoretical properties of the VCM.

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In contrast to Sefton and Steinberg (1996), we are interested in the VCM experiments with QL1 and QL2 using a fixed group setting. Since the fixed group setting transforms the game into a super game, subjects might be motivated to play strategically in such an environment (for details, see the discussion in Sefton and Steinberg (1996)). Furthermore, because the group members are fixed, the feedback from preceding periods contributes to belief formation much more directly in the fixed group setting than it does in the randomly re-matched group setting. Healy (2006) provides experimental evidence that subjects appear to best respond to recent observations in the VCM experiment with QL1 using a fixed group setting.

Recently, Saijo (2014) showed that, if subjects follow the assumptions of self-interested players and myopic best response dynamics, all Nash equilibria are not asymptotically stable in the system of the VCM with QL1.[2] This leads to a pulse of contributions (alternating between contributing nothing and contributing everything). This dynamic analysis predicts that the feedback from repeated trials will worsen the coordination problem in the VCM with QL1. On the other hand, Laury et al. (1999) found that the symmetric Nash equilibrium was a poor predictor of individual contributions and that mean contributions also varied widely among individuals, even within a single experiment. This result was confirmed by Hichri and Kirman (2007). These observations and the instability result suggest a complex interaction among subjects in the VCM with QL1.

Analogous arguments of instability were discussed concerning oligopoly competition in the field of industrial organization (see Cox and Walker, 1998; Rassenti et al., 2000; Huck et al., 2002). Nevertheless, the instability problem in the VCM with QL1 differs from that examined in those discussions. As Andreoni (1995) pointed out, subjects are called upon to generate positive externalities in the VCM environment, whereas they are asked to generate negative externalities in the experiment of oligopoly competition.[3] The positive and negative framing will lead to different effects on cooperation (see Andreoni, 1995; Sonnemans et al., 1998; Cookson, 2000; Bowles and Polania-Reyes, 2012). Cooperative behavior is widely observed in the VCM experiments (for a survey, see Chaudhuri, 2011). Therefore, an investigation in the VCM environment might provide a new understanding of the effect of instability in an environment that includes cooperation.

More importantly, most experimental studies in the field of VCM experiment have used the linear payoff function, which might have failed to capture the real-world instability of the VCM. Therefore, this study investigates that instability and provides dynamic analyses on the convergence of individual contributions in the VCM with QL1 using a fixed group setting. The results of the VCM experiment with QL2 serve as a reference.

In contrast to the observation of a tiny difference in contribution levels between the QL1 and QL2 environments using a randomly rematched group setting in Sefton and Steinberg (1996), our experimental results show a significant difference in the convergence of individual contributions between the QL1 and QL2 environments using a fixed group setting. We find clear evidence that the dispersion of individual contributions decreases, indicating the convergence of individual contributions, and that the absolute changes of individual contributions diminish, suggesting that individual contributions become steady, in the experiment with QL2.[4] Conversely, although we do not find a clearly unstable pulsing in the group's total contribution in the experiments with QL1, our observations show that the dispersion of individual contributions increases progressively with repeated trials and that individual contributions are still volatile in the experiments' last periods. Hence, individual contributions diverge in the QL1 environment. These observations suggest that the coordination problem is not alleviated and that individual contributions are not converging to any equilibrium in the experiments with QL1. Therefore, our main result is that the experimental observations provide supporting evidence for the non-convergence of individual contributions in the QL1 environment using a fixed group setting, but there is still a significant distance between our theoretical instability argument and our experimental observations.

Moreover, consistent with the findings of previous studies, our data show considerable cooperation across players in all experiments. In each experiment, almost 50 percent of the subjects could be considered as following the decision rule of typical conditional cooperators, and about 20 percent of the subjects are weak free riders.[5] Based on this observation, we discuss possible explanations for the distance between our theoretical predictions and the experimental observations in the conclusion.

The remainder of this paper is organized as follows. Section 2 summarizes several theories concerning the VCM with QL1 and QL2. Section 3 presents our experimental design. Section 4 reports the experimental observations. Finally, the last section discusses the results and concludes the study.

2. Theories of the VCM with QL1 and QL2

2.1. VCM with QL1

Suppose that, in an n-player VCM with QL1, all players have the same payoff function and the same endowment E. A simple quadratic specification is the following:

$$\pi_i = E - s_i + aS - bS^2, \tag{1}$$

where a and b are positive constants, s_i denotes player i's individual contribution, and $S = \sum_{i=1}^{n} s_i$ represents the group's total contribution. For this simple game, a list of individual contributions $\hat{s} = (\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_n)$ is a Nash equilibrium if, for all i, $\pi_i(\hat{s}_i, \hat{s}_{-i}) \geq \pi_i(s_i, \hat{s}_{-i})$ for all $s_i \in [0, E]$, where $\hat{s}_{-i} = \sum_{j \neq i} \hat{s}_j$. Therefore, from the first-order condition, the sum of Nash equilibrium contributions is given as

$$\hat{S} = \frac{a-1}{2b}, \ \hat{S} \in [0, \ nE]. \tag{2}$$

This result indicates that any combination of individual contributions constitutes a static Nash equilibrium as long as the total contribution equals $\hat{S}$ (Bergstrom et al., 1986).

Anderson et al. (1998) introduce decision errors into this model. They show that, though there is a continuum of Nash equilibria, a unique logit equilibrium exists that is symmetric across players. The equilibrium density is a (truncated at the boundary of the choice set) normal density for the quadratic public goods game (the VCM with QL1).[6] Furthermore, they suggest that the quadratic model can easily be generalized to allow for individual differences in error parameters. The unique symmetric logit equilibrium thus becomes a unique asymmetric logit equilibrium. Moreover, because the distribution is truncated by the boundary of the choice set, the expected contribution of the logit equilibrium is also sandwiched between the symmetric Nash

[2] An intuitive explanation of asymptotic stability is that an equilibrium $\hat{x}$ is asymptotically stable if all nearby solutions not only stay nearby but also tend to $\hat{x}$ (Hirsch and Smale, 1974, p. 180). We provide the formal definition of asymptotic stability in Section 2.

[3] The VCM experiments usually frame the subject's choice as contributing to the provision of public goods, which could benefit other players within the group, whereas the oligopoly experiments usually frame the subject's choice as providing a product, which will lower the market price and result in a disbenefit to others within the group.

[4] Absolute changes are the absolute values of the first-order differences of individual contributions.

[5] Typical conditional cooperators are those players who always try to match the average contribution of others in the previous period and whose contribution is insignificantly different from the average contribution of others. Weak free riders are those whose contribution is significantly below the average contribution of other players in the group and who are affected by the difference between their individual contributions and the average.

[6] See Proposition 3 in Anderson et al. (1998).

equilibrium level and half of the endowment. These findings seem consistent with the observations of Isaac and Walker (1998). The rationale behind this comparative static analysis is that the feedback from repeated trials will help subjects achieve the equilibrium consistency condition of the logit equilibrium and solve the coordination problem.[7]

However, this comparative static analysis is based on the assumption that the dynamic system of VCM is stable and converging to the unique logit equilibrium. If the equilibrium consistency condition of the logit equilibrium cannot be reached with belief updating, this implies that the system is unstable, and the comparative static analysis might thus not be suitable.

Saijo (2014) explores the equilibrium in the VCM with QL1 based on a dynamic analysis. It is well-known that the best response function in the VCM with QL1 is as follows:

$$s_i = -\sum_{j\neq i} s_j + \widehat{S},\quad s_i \in [0,\, E], \tag{3}$$

where $\widehat{S}$ is the Nash prediction for the aggregate contribution given by Eq. (2) (see Bergstrom et al., 1986). If players simply follow the myopic Cournot best response dynamics, player i's contribution at period t directly responds to the total contribution of others in the group at period t-1. The best response function (3) then becomes

$$s_i^t = -\sum_{j\neq i} s_j^{t-1} + \widehat{S},\quad s_i \in [0,\, E] \tag{4}$$

Now, let us look at the stability property of this dynamic system. We employ the following definition of asymptotic stability.

Definition 1. An equilibrium $\hat{x}$ is locally asymptotically stable, if and only if there exists some open neighborhood O of $\hat{x}$ such that, for any $x^t \in O$, x^t converges to $\hat{x}$ as t approaches infinity.

A useful conclusion concerning whether the Nash equilibria in the difference equations system of Eq. (4) are asymptotically stable is the following property (see Bischi et al., 2009; Saijo, 2014). Let k be the slope of the best response function at the Nash equilibrium.

Property 1. The system $s_i^t = r(s_{-i}^{t-1})$, (i = 1, 2, …, n) is locally asymptotically stable if and only if $|k(n-1)| < 1$.

Since the slope of Eq. (4) is -1 and $n-1 \nless 1$ if $n \geq 2$, all equilibria are not locally asymptotically stable, and contributions will alternate between contributing nothing and contributing everything after a few rounds (if $\widehat{S} \geq E$) in a simultaneous difference equation system of the VCM with QL1 under the assumptions of self-interested subjects and myopic best response. The rationale behind this theoretical result is that the feedback from repeated trials will not alleviate the coordination problem, but worsen it. This insight implies the possibility that the dynamic system of a VCM experiment with QL1 is unstable.

2.2. VCM with QL2

In an n-player VCM with QL2, a simple quadratic payoff function is given as follows:

$$\pi_i = c(E - s_i) - d(E - s_i)^2 + S, \tag{5}$$

where c and d are positive constants. Then, from the first-order condition, a dominant Nash equilibrium solution for every player is given as

$$\hat{s} = \frac{1-c}{2d} + E,\quad \hat{s} \in [0,\ E] \tag{6}$$

Therefore, in the VCM environment with QL2, due to a unique dominant equilibrium, subjects will face a decision environment similar to the VCM with linear payoff functions. The only difference is the location of the equilibrium in the choice set. Anderson et al. (1998) also introduce decision errors into the quadratic model of the VCM with QL2. Since the distribution of the logit equilibrium is also truncated by the boundary of the choice set, they suggest that the decision error should partially explain excessive giving when the Nash equilibrium is less than half of the endowment. Willinger and Ziegelmeyer (2001) provide experimental observations for this theoretical result.

3. Experimental design and procedure

The experiments were conducted at the Vernon Smith Experimental Economics Lab at Shanghai Jiaotong University (SJTU) in March 2015 (192 subjects) and March 2017 (96 subjects). The subjects were SJTU students excluding those from the Department of Economics and Management. All subjects participated voluntarily and had no experience of VCM experiments using nonlinear payoff structures. The experiments consisted of 12 sessions. For each session, we recruited more than 30 subjects. We then used a random mechanism to select the participants. Twenty-four subjects were selected in each session, and we paid a show-up fee to the rest. We used z-Tree to run the experiments (Fischbacher, 2007).

Table 1 shows the parameters of our experimental design.[8] We implement four different experiments. Three of these (QL1N, QL1P, and QL1M) utilize payoffs based on QL1, which is linear with respect to the private good and nonlinear with respect to the public good, while QL2N is based on QL2, which is linear in the public good and nonlinear in the private good.

Following the design of Sefton and Steinberg (1996), we set the following consistency conditions for the two experiments with nonlinear designs (QL1N and QL2N):

1. The same (symmetric) equilibrium contribution of two tokens per individual.
2. The same (symmetric) optimal contribution of six tokens per individual.
3. The approximately equal reward from (symmetric) equilibrium play.
4. The approximately equal reward from (symmetric) socially optimal play.[9]

However, our experimental design differs from that of Sefton and Steinberg (1996) in two key ways. First, in our design, eight subjects are randomly allocated to a group at the beginning of the experiment. Their positions are fixed throughout the experiment. In the design of Sefton and Steinberg (1996), four individuals are randomly allocated to a group at the beginning of each period. We use a relatively large group, following Ostrom et al. (1992), who use an eight-player group setting to study common pool resource environments.

Second, since assuming the coefficient of linear parts to be equal to one could make it easy for subjects to understand the nonlinear return structures in payoff tables, we do not consider the 5th symmetric condition in Sefton and Steinberg (1996)—the same monetary loss from a one-token unilateral departure from equilibrium play. The result is that the opportunity cost among choices in the QL1N experiment is significantly lower than that in the QL2N experiment. As Smith and Walker (1993) have shown, the opportunity cost among choices is directly related to the dispersion of individual choices in experiments. Therefore, the relatively small opportunity costs might affect the

[7] The equilibrium consistency condition is that player i's expectations of other players' actions are equal to the means of the actual equilibrium distributions (Anderson et al., 1998).

[8] Payoff lists and instructions translated from the Chinese version can be found among the supplementary documents. We also present graphs for the relation between returns and tokens for each account and clearly display which part indicates diminishing marginal returns. This makes our design close to the DET experiments in Laury et al. (1999).

[9] We set an additional payment to make the rewards from equilibrium play and socially optimal play approximately equal between the two experiments.

Table 1
Parameters of the experiments.

Experiments	QL1N	QL1P	QL1M	QL2N
Payoff function	QL1N: $(E - s_i) + 1.4484S - 0.0137(S)^2 + 28$ QL1P and QL1M: $\begin{cases} 10(E - s_i) + 15S, & S \le 16; \\ 10(E - s_i) + 5(S - 16) + 240, & 16 < S \le 48; \\ 10(E - s_i) + (S - 48) + 400, & 48 < S \le 64. \end{cases}$ QL2N: $11.5(E - s_i) - 0.875(E - s_i)^2 + S$			
Endowment (Tokens)	8	8	8	8
Additional payment (E$) (symmetric)	28	0	0	0
Nash choice $\hat{s}$ (Payoff) (symmetric)	2(53.7)	2(300)	2(300)	2(53.5)
Socially optimal s^*(Payoff)	6(68)	6(420)	6(420)	6(67.5)
Payment ratio	22:1	110:1	110:1	22:1
Periods	15(Random ending)	30	30	15(Random ending)
Groups/Subjects	12/96	6/48	6/48	12/96

s_i denotes the individual contribution of player i; E represents the endowments; and S denotes the group's total contribution.

convergence of choices.[10] To ensure that our experimental observations do not originate from the relatively small opportunity costs, we design the other two experiments (QL1P and QL1M) for robustness checks. The QL1P experiment employs a piecewise linear payoff function as the linear approximation for the nonlinear returns from the public good (see also, the payoff design in Cason and Gangadharan, 2014). We also increase the opportunity costs among choices.[11] The QL1M experiment uses the same payoff function as that used in the QL1P experiment but with a different framing of the payoff table in the instructions. The new payoff table uses a matrix to directly connect the choices to the payoffs (see, e.g., the design of payoff tables in Cason et al., 2004).

To clearly illustrate the stability property of our design, we draw the best response curves for the two environments in Fig. 1. In this figure, the horizontal axis is the total contribution of others in the group, and the vertical axis represents player i's own contribution. For the three experiments with QL1, the myopic Cournot response curve (the bold black line "f-w-j-h") is

$$s_i^t = \min\left\{\max\left\{-\sum_{j\neq i} s_j^{t-1} + 16,\ 0\right\},\ 8\right\}. \tag{7}$$

Consider an example. Suppose that every player's initial contribution is the same at a/7, which implies that the total contribution of others is initially "a." Obviously, the best response to "a" is point "b." Then, the total contribution of others goes to "c." Then, we find the best response to "b" is point "d," that to "d" is point "f," and that to "f" is point "h." Finally, the dynamic difference system will be pulsing between point "f" and point "h." This example shows that the contributions of subjects will be pulsing between 0 and 8 after a few rounds. However, for the QL2N experiment, this curve is derived simply as follows:

$$s_i^t = 2. \tag{8}$$

Since the best response curve is flat, the best response to any case is contributing two tokens. Given these theoretical results, we propose the following hypotheses:

Hypothesis 1. In the experiment with QL2 (QL2N), individual contributions will converge to the unique Nash equilibrium, which indicates that (*i*) the dispersion of individual contributions decreases and (*ii*) individual contributions become steady with repeated trials.

Hypothesis 2. In the experiments with QL1 (QL1N, QL1P, and QL1M), individual contributions will not converge to the symmetric and asymmetric Nash equilibria, which indicates that (i) the group's total contribution will be pulsing round after round (the sample autocorrelation statistic should be negative), (ii) the dispersion of individual contributions might not decrease because of the intergroup level heterogeneity, and (iii) individual contributions will be volatile even in the last periods.

For each session in the QL1N and QL2N experiments, we employed a random ending rule. Subjects were certain to participate in the first 15 periods. From the beginning of the 16th period, the experiment would continue with a probability of 0.3. This setting helped to suppress strategic play (e.g., the endgame effect) in a repeated game with the fixed group setting.[12] Data from the first 15 rounds were used for analysis. Furthermore, to give more information regarding the convergence of contributing behavior, the public goods game repeated 30 periods for each session in the QL1P and QL1M experiments. Since these two experiments serve as robustness checks for the observations from the QL1N experiment, we have the third hypothesis.

Hypothesis 3. The dynamic patterns of contributions (concerning dispersion and contribution volatility) should not be significantly different among the QL1N, QL1P and QL1M experiments.

At the beginning of each period, each subject received eight tokens. They were called upon to allocate these tokens to two accounts: the private account and the public account. All tokens had to be allocated in each period without communication with others, and the feasible choice set was $\{0, 1, \ldots, 7, 8\}$. Each token in the private account would produce a private return to oneself. Each token in the public account would produce a public return to each member of the group. The framing of instructions was similar to that of Sefton and Steinberg (1996) and consistent across experiments.

At the end of each period, the result was reported to each subject. The report consisted of three parts: each subject's own decisions, the total tokens in the public account, and his/her own payoff. No subject could observe the individual contributions of other members of the same group. This incomplete information setting is consistent with most of the literature on VCM experiments.

When all 24 subjects entered the lab, the instructions were distributed to each one. A native speaking research assistant read the instruction loudly. Then, control questions were required to be answered correctly to ensure that every subject understood the experimental procedure. At the end of the experiment, each subject received his/her payment privately at a preannounced exchange rate of 22 experimental dollars (E$) to 1 Chinese RMB in the QL1N and QL2N experiments and 110 experimental dollars (E$) to 1 Chinese RMB in the QL1P and QL1M experiments. The 192 subjects earned RMB 44.5 (7.5 US dollars) each on average, with a range of RMB 36 to RMB 47 in the QL1N and QL2N experiments. The 96 subjects earned RMB 94 (15 US dollars) each on average, with a range of RMB 80 to RMB 108 in the QL1P and QL1M experiments. Each session lasted about one hour and a half, including the instruction and payment distribution time.

[10] We thank an anonymous referee for pointing this out.

[11] Different from the QL1N experiment, we remove the fixed payment in each period and boost the magnitude of experimental payoffs by 10 times, but the exchange ratio from experimental dollars to real money increases by only five times (from 22:1 to 110:1) in the QL1P and QL1M experiments. For the choices around the Nash equilibrium, the opportunity cost in the piecewise linear design is significantly greater than is that in the nonlinear design.

[12] See Dal B´o (2005). However, other studies find no significant difference between the finite period setting and the random terminated setting (e.g., Selten and Stoecker, 1986; Engle-Warnick and Slonim, 2004).

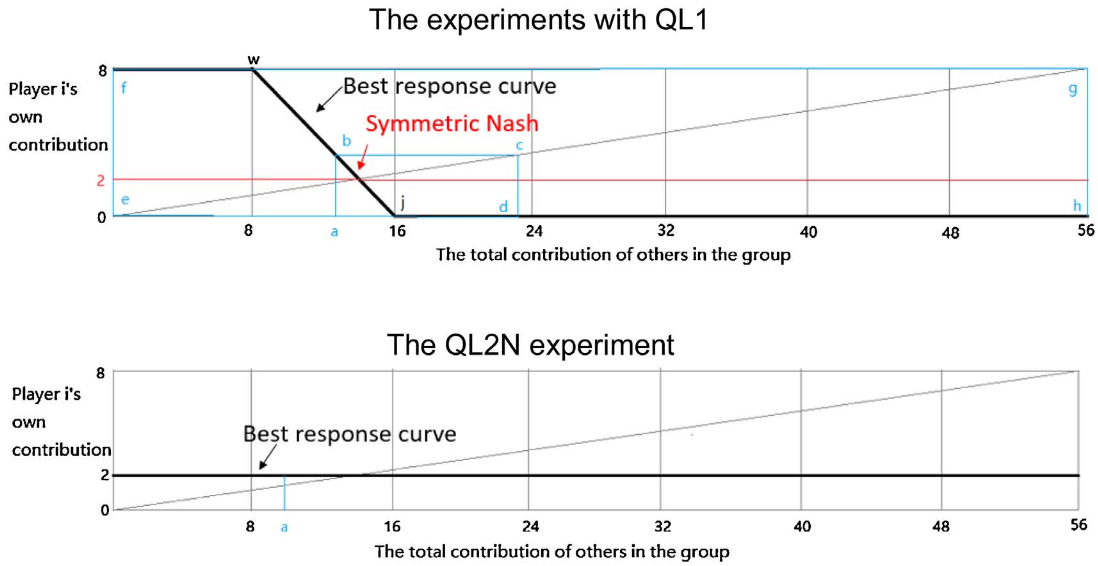

Fig. 1. Stability property of the design.

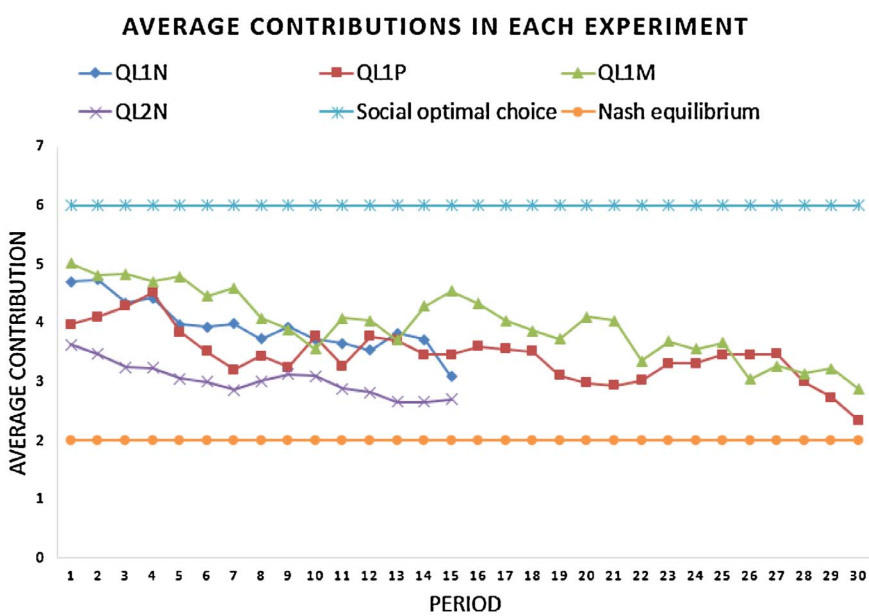

Fig. 2. Average contributions in the four experiments.

4. Results

This section consists of four subsections. The first reviews the experimental data. The second investigates the dispersion of individual contributions. The third shows the tendency of changes in individual contributions. The final subsection investigates the conditional cooperation in the four experiments and roughly classifies subjects.

4.1. Overview

First, we present an overview of the contributions. Fig. 2 shows the average contributions to the group account at each period for the four experiments. A decreasing tendency of average contributions is shared by the four experiments. The Wilcoxon signed-rank tests show that individual contributions from periods 11 to 15 are significantly lower than those from periods 1 to 5 in both the QL1N and QL2N experiments (p-values = 0.0000) and that individual contributions from periods 21 to 30 are significantly lower than those from periods 1 to 10 in both the QL1P (p-value = 0.0171) and QL1M (p-value = 0.0000) experiments.[13]

Time series plots of the group's total contribution are provided in Fig. 3. The total contributions of all groups are significantly above the Nash prediction, indicating the presence of cooperation. Sample autocorrelation statistics (α) of the groups' total contributions, reported in Table 2, are positive for all groups. The Wilcoxon rank-sum test shows a slight difference in autocorrelation statistics between the QL1N and QL2N experiments (p-value = 0.0781). Fig. 4 shows that the group's total contribution is pulsing more in some groups in the QL1N experiment than in the QL2N experiment. However, the unstable pulsing seems to have been considerably smoothed compared to the prediction of instability in Saijo (2014), whereby it should generate a negative

[13] For all Wilcoxon signed-rank tests in this paper, we first compute two averages across periods 1 to 5 and 11 to 15 for each subject in the QL1N and QL2N experiments and across periods 1 to 10 and 21 to 30 for each subject in the QL1P and QL1M experiments. Then, we conduct the Wilcoxon signed-rank tests over two samples of averages to eliminate correlation across periods.

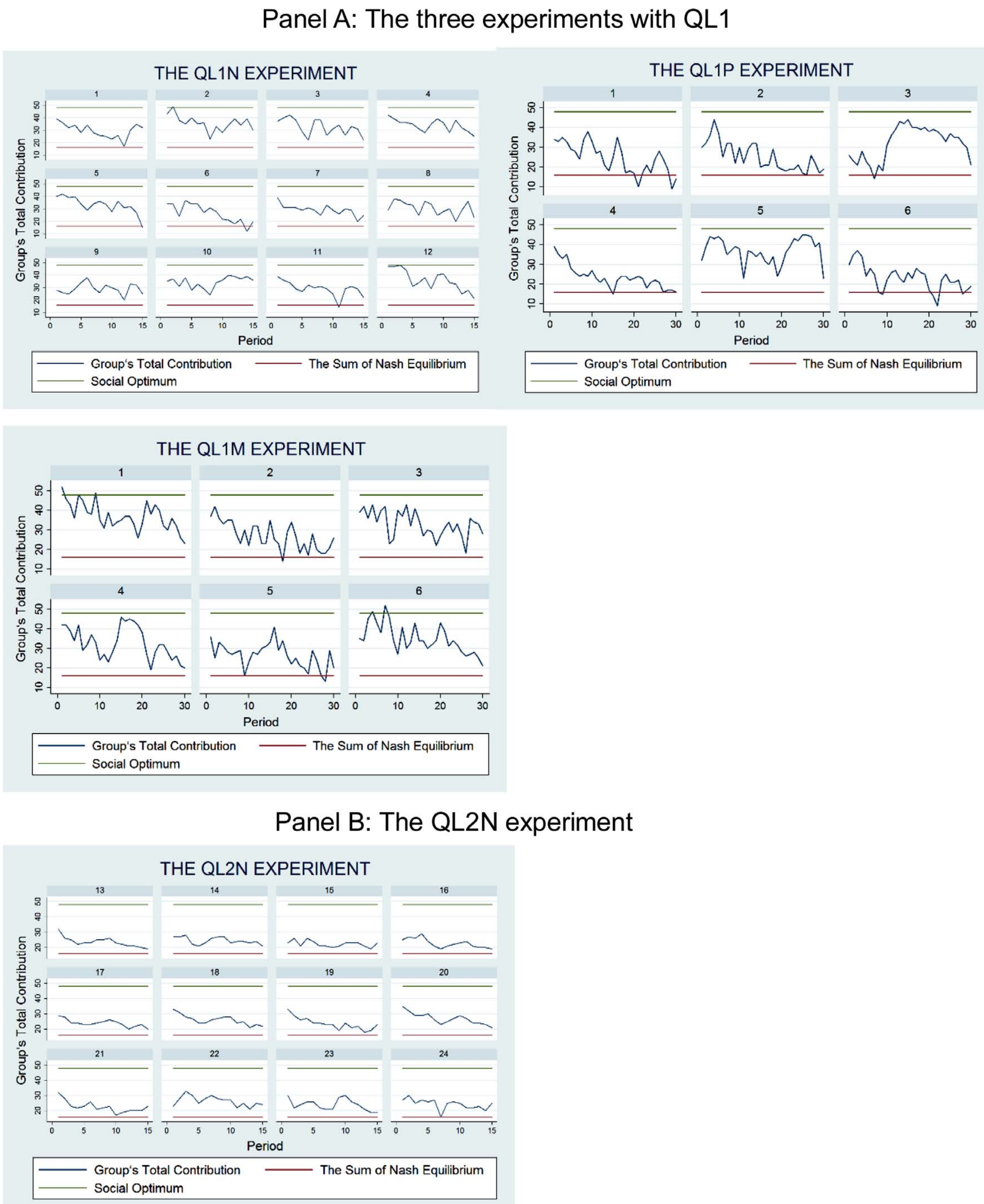

Fig. 3. Time series plots of groups' total contributions.

serial correlation in the experiments with QL1. These observations reject the first prediction of hypothesis 2.

4.2. Dispersion

Next, we show the dynamics of dispersion in the four experiments. A common way to do this in statistics is using the coefficient of variation to compare dispersion between two samples with different averages. However, we focus on the dispersion of choices rather than the dispersion of numbers. In this context, each number of contributions represents each position of actions in the choice set. Here, two contribution samples of {0,0,1,1,2,2,3,3} and {5,5,6,6,7,7,8,8} share an identical dispersion although their averages are different. Therefore, we still use the standard deviation as a measure of dispersion.

Result 1 (**Dispersion**): Although average contributions are declining in all four experiments, the standard deviation of individual contributions is ascending in the three experiments with QL1 at the aggregate level, whereas it is descending in the QL2N experiment. The ascending standard deviation of individual contributions at the aggregate level stems from the intragroup level in the three experiments with QL1.

Support: Fig. 4 shows the standard deviations of individual contributions at each period for the four experiments. At the beginning of the experiment, the standard deviations of the four experiments are close. However, the Spearman's rank correlation tests reveal an ascending tendency shared by the three experiments with QL1 ($\rho = 0.7857$, p-value < 0.001 for QL1N; $\rho = 0.7130$, p-value < 0.001

Table 2
Sample autocorrelation statistics.

The QL1N experiment

Group	1	2	3	4	5	6	7	8	9	10	11	12
α	0.43	0.27	0.21	0.39	0.55	0.27	0.18	0.11	0.03	0.33	0.53	0.34

The QL1P experiment

Group	1	2	3	4	5	6
α	0.68	0.62	0.83	0.71	0.41	0.65

The QL1M experiment

Group	1	2	3	4	5	6
α	0.42	0.47	0.29	0.68	0.35	0.53

The QL2N experiment

Group	1	2	3	4	5	6	7	8	9	10	11	12
α	0.45	0.37	0.10	0.65	0.48	0.56	0.53	0.57	0.45	0.38	0.36	0.09

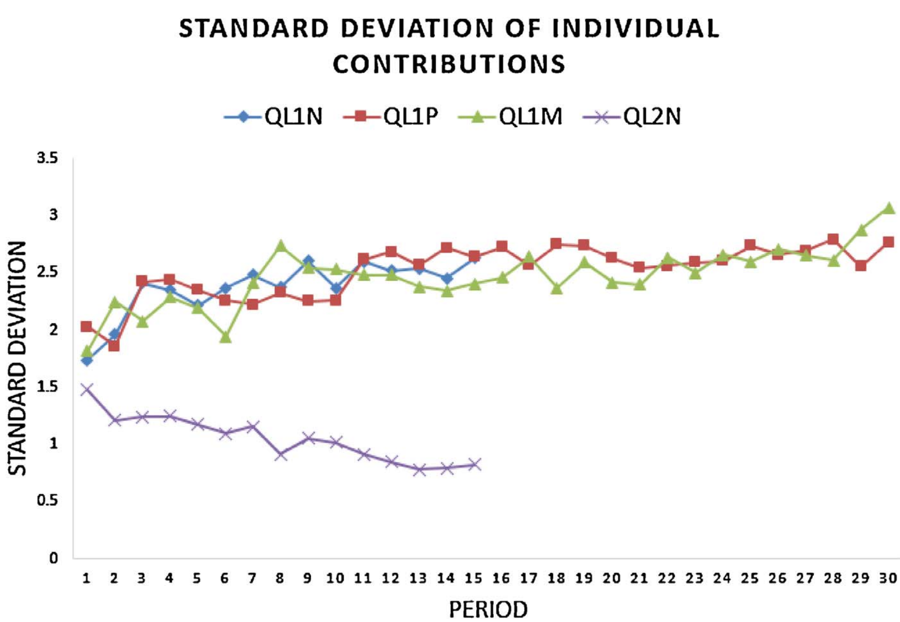

Fig. 4. Standard deviation of individual contributions.

for QL1P; $\rho = 0.7433$, p-value < 0.001 for QL1M), yet a descending tendency appears in the QL2N experiment ($\rho = -0.9464$, p-value < 0.001).

Time series plots of the standard deviation for each group in the four experiments are provided in Fig. 5. In the three experiments with QL1, the Spearman's rank correlation tests show that eight out of 12 groups from the QL1N experiment, three out of six groups from the QL1P experiment, and five out of six groups from the QL1M experiment share a significantly increasing pattern (p-values < 0.1 for 16 groups; p-values < 0.05 for 11 groups); and no group shows a significantly decreasing pattern. By contrast, eight out of 12 groups share a significantly decreasing pattern (p-values < 0.05), and no group shows a significantly increasing pattern in the QL2N experiment.

To sum up, the observation that the standard deviation of individual contributions is ascending at the aggregate level stems from the intragroup level in the three experiments with QL1. These observations do not support that individual contributions are converging to a symmetric equilibrium in the experiments with QL1. However, we also notice that the increasing dispersion at the aggregate level stems mainly from the intragroup level rather than the intergroup level.[14] This observation is inconsistent with the reasoning of our instability argument.

Therefore, Result 1 supports the first prediction of hypothesis 1, but rejects the reasoning of the second prediction of hypothesis 2. Furthermore, the observation that all the three experiments with QL1 share similar dynamics of dispersion supports hypothesis 3.

4.3. Absolute changes in individual contribution

We use the absolute value of the first-order difference of individual contributions ($s_i^t - s_i^{t-1}$, $t \geq 2$; hereafter "AVFD") to measure the

[14] We also check the dynamical tendency of the standard deviation of the group's total contributions across periods in the four experiments. The Spearman's rank correlation tests show that $\rho = 0.2536$ and p-value $= 0.3618$ for QL1N, $\rho = 0.5537$ and p-value $= 0.0015$ for QL1P, $\rho = 0.0007$ and p-value $= 0.9972$ for QL1M, and $\rho = -0.6643$ and p-value $= 0.0069$ for QL2N. These results indicate that, in two of the three experiments with QL1, the dispersion at the intergroup level does not increase with repeated trials.

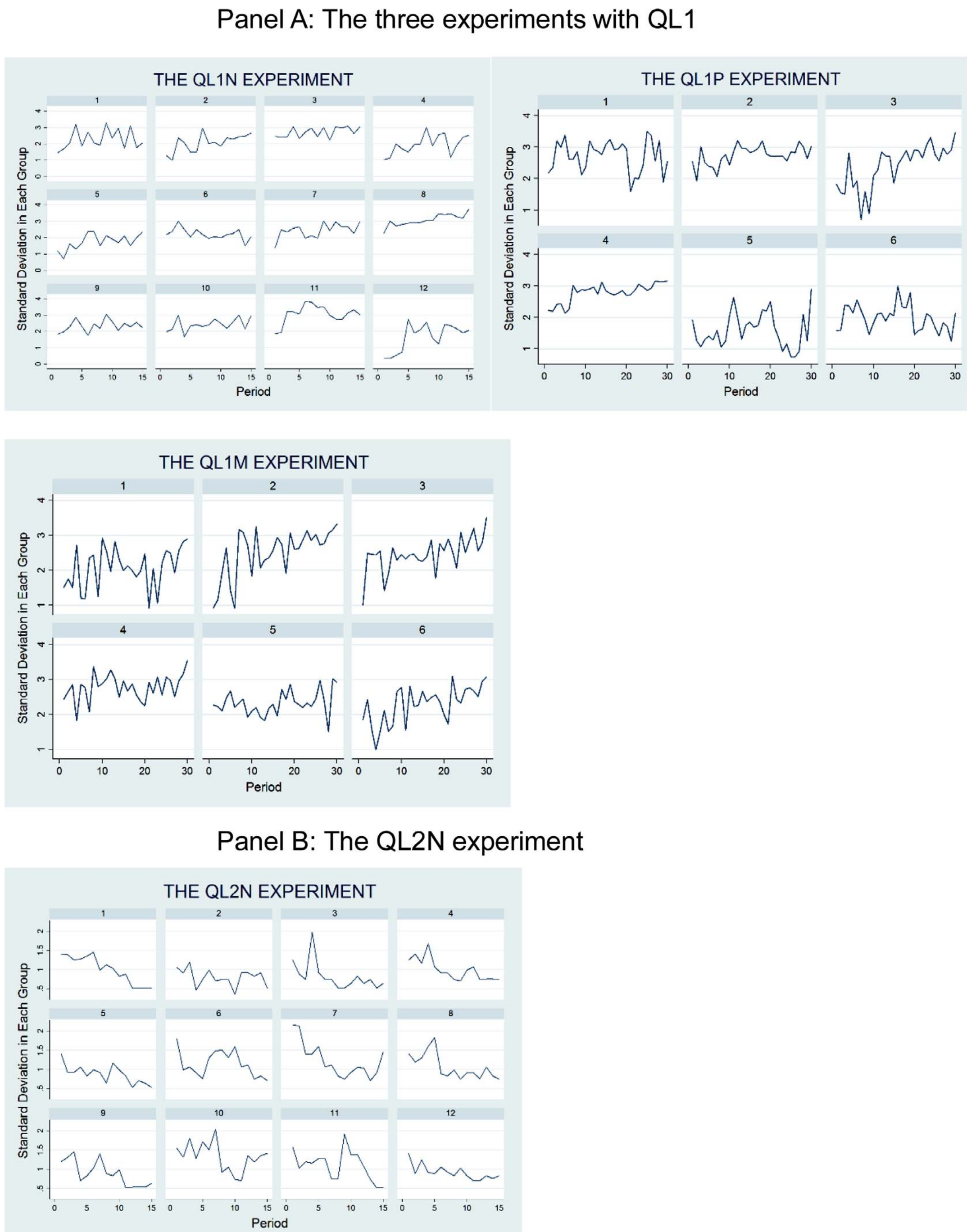

Fig. 5. Time series plots of standard deviations in groups.

pulsing of individual contributions. If the system is approaching an equilibrium, the degree of contribution pulsing on average will diminish.

Result 2 (**Absolute changes**): The absolute changes on average are diminishing in the QL1P and QL2N experiments. In the QL1N and QL1M experiments, however, they do not diminish relative to the beginning of the experiment.

Support: Fig. 6 shows the average of AVFDs at each period for the four experiments. By comparing sample 1 (the AVFDs from periods 2 to 6) with sample 2 (the AVFDs from periods 11 to 15), the Wilcoxon signed-rank test shows a significant decrease in the QL2N experiment (p-value = 0.0000), but an insignificant result for the QL1N experiment (p-value = 0.1312). Furthermore, for the QL1P and QL1M experiments, by comparing sample 1 (the AVFDs from periods 2 to 11) with sample 2 (the AVFDs from periods 21 to 30), the Wilcoxon signed-rank test shows a significant decrease in the QL1P experiment (p-value = 0.0012) yet an insignificant result for the QL1M experiment (p-value = 0.4817). Although there is also a decreasing tendency in the QL1P experiment, the AVFDs in the last 10 periods of the QL1P experiment are still significantly greater than those in the last five periods of the QL2N experiment (p-value = 0.0124, by the Wilcoxon rank-sum test).

Combined with previous observations of standard deviations, the decreasing AVFDs in the QL2N experiment indicate that the experimental system is converging to the dominant equilibrium, which is symmetric across players. Conversely, the decreasing AVFDs in the QL1P experiment might indicate that some groups in the experiments with QL1 are converging to some asymmetric equilibrium. Therefore, we further check the AVFDs at the group level. Comparing sample 1

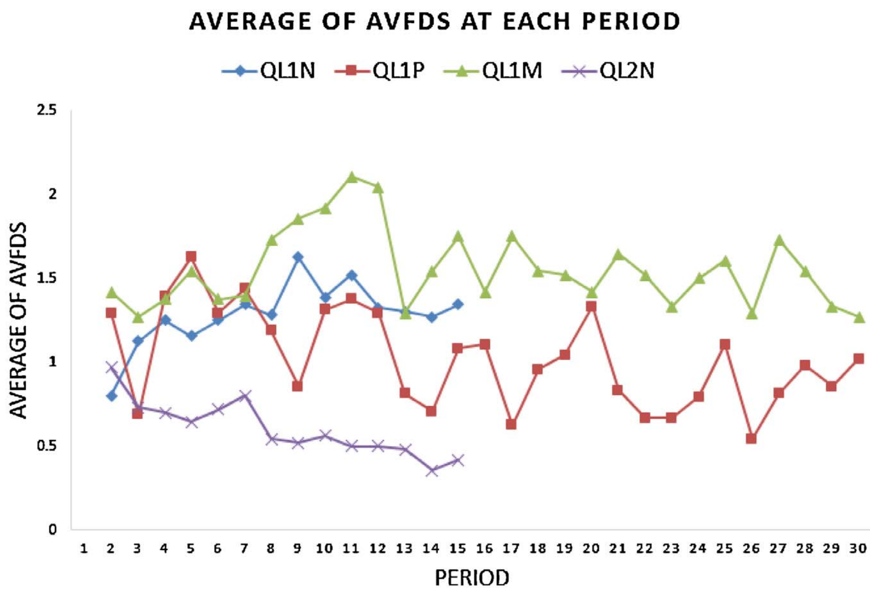

Fig. 6. Average of AVFDs at each period.

with sample 2 in each group of the three experiments with QL1 reveals a significant decrease in four groups (p-value = 0.0138 for group 10 in the QL1N experiment; p-value = 0.0117 for group 2 and p-value = 0.0687 for group 4 in the QL1P experiment; and p-value = 0.0929 for group 1 in the QL1M experiment). However, by checking the individual data in these four groups, we find that the individual contributions from a part of the group members are still volatile in the last periods of the experiment. This is not compatible with the experimental system's converging to a static asymmetric equilibrium.

Therefore, Result 2 supports the second prediction of hypothesis 1 and the group level observations also support the third prediction of hypothesis 2. Furthermore, although the observation in the QL1P experiment at the aggregate level is different from those in the other two experiments with QL1, the group level observations show that individual contributions are volatile in the last periods of all the three experiments with QL1, which is consistent with the prediction of hypothesis 3.

Overall, our experimental data reveal a clear pattern showing that contributions are converging to the static equilibrium in the QL2N experiment. By contrast, our observations do not suggest the existence of a process whereby the dynamic system is approaching a symmetric or asymmetric equilibrium and that the coordination problem is alleviated in the three experiments with QL1. However, we also notice that there is not a significant pulsing in the group's total contributions in the three experiments with QL1 and that the increasing dispersion of individual contribution comes mainly from the intragroup level. Our instability theory cannot explain these observations. Therefore, in the following subsection, we investigate the heterogeneity among individuals in order to generate insights concerning these observations through a categorization of the subjects.

4.4. Conditional cooperation

In the VCM experiments with linear payoff functions, players are often divided into several categories. The three most common categories are free riders, conditional cooperators, and unconditional cooperators. Free riders account for only around 20 percent of the total population. However, conditional cooperators account for around 50 percent (see Fischbacher et al., 2001; Sonnemans et al., 1999; Keser and van Winden, 2000; for a survey, see Chaudhuri, 2011). These findings indicate that the experimental environment might be much more complex than the assumption in Saijo (2014) implies. For the QL1 environment, since Laury et al. (1999) found that average contributions varied widely among individuals, even within a single experiment, there might be a systematic difference in the motivation for cooperation between the experiments with QL1 and QL2.[15] In this subsection, we attempt to investigate the conditional cooperation from a myopic perspective to see whether there is a systematic difference in conditional cooperation across the experiments.

The individual decision rule, used to isolate the motivation of conditional cooperation, is assumed to take the following form.

$$s_i^t - s_i^{t-1} = \alpha_i + \beta_i\left(s_i^{t-1} - \frac{1}{7}\sum_{j\neq i} s_j^{t-1}\right) + \varepsilon_i,\quad t \geq 2, \tag{9}$$

where ε_i is the residual term of player i. Eq. (9) is estimated using the Seemingly Unrelated Regressions (SUR) method for each group of eight players in the four experiments. In this regression, $-\frac{\alpha_i}{\beta_i}$ approximately denotes the overall distance between player i's contribution and the average contribution of other players in the group. Thus, this regression allows us to check two aspects of the subjects' contribution behavior: first, how many players are reacting to the difference between their own contribution and the average contribution of others (or how many players try to match the average contribution of others in the previous period); second, the overall distance between player i's contribution and the average contribution of other players. If $\alpha_i > 0$ and $\beta_i < 0$, subject i's contribution is significantly above the average contribution of other players in the group and is also affected by the difference between his/her contribution and the average. This result indicate that this subject is a *weak* unconditional cooperator (WUC).[16] In turn, if $\alpha_i < 0$ and $\beta_i < 0$, a *weak* free rider (WFR) is indicated. A *typical* conditional cooperator (TCC) should have $\alpha_i = 0$ and $\beta_i < 0$, which implies a person who always tries to match the average contribution of others in the previous period and whose contribution is insignificantly different from the average contribution of others. Moreover, unconditional cooperators (UC) are those who persisted in contributing a fixed number of at least six tokens; conversely, free riders (FR) are those who persisted in contributing a fixed number of no more than two tokens.

[15] Here, the term "systematic difference in the motivation for cooperation" is used to indicate the difference in the distribution among different types of subjects.

[16] We call them "*weak* unconditional cooperators" to distinguish them from those unconditional cooperators who always contribute six tokens throughout the experiment.

Table 3
Conditional cooperation.

Form	$s_i^t - s_i^{t-1} = \alpha_i + \beta_i\left(s_i^{t-1} - \frac{1}{7}\sum_{j\neq i} s_j^{t-1}\right) + \varepsilon_i,\ \ t \geq 2$			
Individual results				
Category	QL1N (96 subjects)	QL1P (48 subjects)	QL1M (48 subjects)	QL2N (96 subjects)
UC	3 (3%)[b]	1 (2%)	2 (4%)	1 (1%)
WUC ($\alpha_i > 0$ and $\beta_i < 0$[a])	19 (20%)	11 (21%)	7 (15%)	17 (18%)
TCC ($\alpha_i = 0$ and $\beta_i < 0$)	40 (42%)	21 (44%)	27 (56%)	48 (50%)
WFR ($\alpha_i < 0$ and $\beta_i < 0$)	21 (22%)	11 (23%)	11 (23%)	20 (21%)
FR	1 (1%)	0 (0%)	0 (0%)	5 (5%)
Unclassified ($\alpha_i = 0$ and $\beta_i = 0$)	12 (12%)	4 (8%)	1 (2%)	5 (5%)

[a] Both α_i and β_i of individual regressions (SUR) are judged by a two-tailed test at the 5% significance level.
[b] Percentages of the total population are reported in parentheses.

Hence, By examining α_i and β_i, we can roughly divide all subjects into six categories.[17]

Result 3 (**Conditional cooperation**): No systematic difference in conditional cooperation is observed across the four experiments. The individual estimates from the SUR show that around 50 percent of the players could be categorized as typical conditional cooperators; weak free riders and weak unconditional cooperators each account for about 20 percent of the total population in all experiments.

Support: Table 3 summarizes the results of the individual regressions. Briefly, by comparing the number of subjects in each type, we find no systematic difference in conditional cooperation across the four experiments. In all experiments, almost half of the players could be regarded as typical conditional cooperators, while weak free riders and weak unconditional cooperators each account for about 20 percent of the total population. This result is consistent with the findings in the linear environment of the VCM experiment. The presence of conditional cooperators might be a reason for the smoothed pulsing in the group's total contribution in the experiments with QL1.

5. Discussion and conclusion

In this study, we conducted experiments to investigate the dynamic pattern of contributing behavior in the VCM with two quasi-linear payoff functions. We find clear evidence that the system is converging to the dominant equilibrium in the QL2N experiment. The average contribution decreases with repeated trials and individual contributions converge and become steady. By contrast, in the experiments with QL1, although contributions on average are also decreasing with no clearly unstable pulsing in the group's total contribution, individual contributions diverge and change continuously.

These observations do not support the hypothesis that the system of the VCM with QL1 is converging to an equilibrium, indicating that a comparative static analysis alone might not be suitable for the VCM with QL1 using a fixed group setting. On the other hand, our observation is consistent with the finding of previous studies on the VCM experiment with linear payoff functions that most players in the lab VCM experiment follow the decision rule of conditional cooperators. This might be a reason for the growing dispersion in the three experiments with QL1.

Considering a repeated VCM game with two types of players—free riders and conditional cooperators—if the game has a dominant strategy, such as that of a linear environment, the decay of the average contribution could be explained by the classical scenario of the interaction between free riders and conditional cooperators. Once the conditional cooperators become frustrated by free riding, they start reducing their contributions. Then, the average contribution becomes close to the dominant equilibrium. Our experimental evidence suggest that this may also be true in the VCM with QL2 in which there is a dominant equilibrium.

The observations of the dispersion and the absolute changes in the three experiments with QL1 indirectly suggest another possible interpretation of the interaction between free riders and conditional cooperators in the VCM with QL1. When the conditional cooperators become frustrated by free riding, they will reduce their contributions to a certain level. The free riders would then have to increase their contribution to increase their payoffs if they expect that the total contribution of others will become less than the sum of the Nash equilibrium contributions. When conditional cooperators find that the total contribution is increasing, they will seek to sustain this total contribution level. However, the free riders will then begin to free ride again, and a new round of decreasing total contribution will begin. We thus conjecture that starting from the dynamic analysis of Saijo (2014) and incorporating the interaction between several different types of players might offer insights into the ascending dispersion we observed in this study.

Finally, two empirical implications of our experimental observations are worth mentioning. First, the experimental observation of the growing dispersion indicates that the stability property of the mechanism itself might also be a reason for the diversity of individual contributions, in addition to the social preference heterogeneity among the players. Second, and more importantly, the experimental observation of non-convergence indicates that the Nash equilibrium might not be a suitable theoretical benchmark to use in empirical analyses of the real-world VCM environment if the system is not converging to it.

[17] There is one subject from the QL1N experiment who should be classified as $\alpha_i < 0$ and $\beta_i = 0$. Because p-value = 0.049 for $\alpha_i < 0$ and only one observation is considered, we take this observation as an unimportant exception and assign this subject into category $\alpha_i = 0$ and $\beta_i = 0$.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.socec.2017.12.002.

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